askIITians

... Powered By IITians

AskIITians IIT JEE Maths Test

Code – AM208

Time - One hour

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

A. General :

1. This booklet is your Question paper containing 69 questions.
2. Blank papers, clipboard, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets in any form are not allowed to be carried inside the examination hall.
3. The answer sheet, a machine-readable Objective Response Sheet (ORS), is provided separately.

B. Filling the ORS :

4. On the lower part of the ORS, write in ink, your name, your Registration No. Do not write these anywhere else.
5. Make sure the CODE on the ORS is the same as that on this booklet and put your signature on the ORS affirming that you have verified.
6. Write your Registration No. in ink, provided in the lower part of the ORS and darken the appropriate bubble UNDER each digit of your Registration No. with a good quality HB pencil.

C. Question paper format.

7. The question paper consists of 3 parts (Physics, Chemistry and Mathematics). Each part has 4 sections.
8. Section I contains 6 multiple choice question. Each question has four choices (A), (B), (C) and (D), out of which only one is correct.
9. Section II contains 4 questions. Each question has four choices (A), (B), (C) and (D), out of which one or more choices is correct.
10. Section III contains 4 questions. Each question contains Statement -1 (Assertion) and Statement -2 (Reason).
    Bubble (A) if both the statements are TRUE and STATEMENT-2 is the correct explanation of STATEMENT-1.
    Bubble (B) if both the statements are TRUE butSTATEMENT-2 is NOT the correct explanation of STATEMENT-2.
    Bubble (C) if STATEMENT-1 is TRUE and STATEMENT-2 is FALSE.
    Bubble (D) if STATEMENT-1 is FALSE and STATEMENT-2 is TRUE.
11. Section IV contains 3 paragraphs. Based upon each paragraph. Three multiple choice questions have to be answered. Each question has four choices (A) (B) (C) (D) out of which only one is correct.

D. Marking Scheme.

12. For each question in Section I, you will be awarded 3 marks if you have darkened only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, minus one (–1) mark will be awarded.
13. For each question in Section II, you will be awarded 4 marks, if you darken only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, (–1) mark will be awarded.
14. For each question in Section III, you will be awarded 3 marks, if you darken only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, (–1) mark will be awarded.
15. For each question in Section IV, you will be awarded 3 marks, if you darken only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, (–1) will be awarded.

Useful Data

Gas Constant	R	= 8.314 J K^{-1} mol^{-1}	**1 Faraday**	=	96500 Coulomb
		= 0.0821 Lit atm K^{-1} mol^{-1}	**1 calorie**	=	4.2 Joule
		= 1.987 ≈ 2 Cal K^{-1} mol^{-1}	**1 Ev**	=	1.6 × 10^{-1} J
Avogadro's Number	Na	= 6.023 × 1023			

Planck's constant	h	$= 6.625 \times 10^{-34}$ J . s $= 6.625 \times 10^{-27}$ erg . s
Atomic No:		H = 1, D = 1, Li = 3, Na = 11, K = 19, Rb = 37, Cs = 55, F = 9, Ca = 20, He = 20, He = 2, O = 8, Au = 79, Ni = 28, Zn = 30, Cu = 29, Cl = 17, Br = 35, Cr = 24, Mn = 25, Fe = 26, S = 16, P = 15, C = 6, N = 7, Ag = 47.
Atomic Masses:		He = 4, Mg = 24, C = 12, O = 16, N = 14, P = 31, Br = 80, Cu = 63.5, Fe = 56, Mn = 55, Pb = 207, Au = 197, Ag = 108, F = 19, H = 1, Cl = 35.5, Sn = 118.6, Na = 23, D = 2, Cr = 52, K = 39, Ca = 40, Li = 7, Be = 4, Al = 27, S = 32.

SECTION – I

1. If a, b, c, d are +ve real no. such that a + b + c + d = 2, then M = (a + b) (c + d) satisfies the relation

   (a) $0 < M \leq 1$ (b) $1 \leq M \leq 2$
   (c) $2 \leq M \leq 3$ (d) none of these

2. If $a^x = b$, $b^y = c$, $c^z = a$, then the value of xyz is
   (a) 0 (b) 1 (c) 2 (d) abc

3. The no. of values of the pair (a, b) for which
   $a(x + a)^2 + b(x^2 - 3x + 2) + x + 1 = 0$ is an identity in x is
   (a) 0 (b) 1
   (c) 2 (d) infinite

4. Which of the following statement is correct?
   (a) $\sin 1^0 > \sin 1$ (b) $\sin 1^0 < \sin 1$
   (c) $\sin 1^0 = \sin 1$ (d) $\sin 1^0 = \frac{\pi}{180} \sin 1$

5. The number of solutions of the equation $\frac{x}{100} = \sin x$ is
   (a) 0 (b) 33
   (c) 63 (d) none of these

6. If $(cot^{-1} x)2 - 5\, cot^{-1} x + 6 > 0$, then $x \in$
   (a) (cot 3, cot 2) (b) (cot 2, ∞)
   (c) (−∞, cot 3) (d) (−∞, cot 3) v (cot 2, ∞)

7. If the median AD of a triangle ABC makes an angle $\propto$ with AB, then sin (A – $\propto$) is equal to

(a) $\frac{b \sin \propto}{c}$ (b) $\frac{b}{b \sin \propto}$

(c) $\frac{c \sin \propto}{b}$ (d) none of these

8. The equation of the bisector of the acute angle between the lines 3x – 4y + 7 = 0 and 12x + 5y – 2 = 0 is

(a) 21x + 7y – 101 = 0 (b) 11x + 3y + 20 = 0
(c) 21x – 7y + 3 = 0 (d) 11x – 3y + 9 = 0

9. Equation of the normal to the circle $x^2 + y^2 – 2ax = 0$ at the point {a (1 + cos$\propto$), a sin$\propto$} is given by:-

(a) y = (x – a) tan $\propto$ (b) y = (x + a) cot $\propto$
(c) y = x tan $\propto$ + a cot $\propto$ (d) none of these

SECTION – II

---

1. Assertion : for any three complex no.s z_1, z_2, z_3 if

$$\Delta = \begin{vmatrix} 1 & z_1 & \bar{z}_1 \\ 1 & z_2 & \bar{z}_2 \\ 1 & z_3 & \bar{z}_3 \end{vmatrix}$$, then

Δ is purely imaginary.

Reason : $\Delta = \sum(z_2\,\bar{z}_3 - \bar{z}_2\,\bar{z}_3)$ and $z - \bar{z}$ is purely imaginary.

2. Assertion : For $n \in N$, $(n!)^3 < n^n \left(\frac{n+1}{2}\right)^{2n}$

Reason : Product of n successive natural numbers is divisible by n!

3. **Assertion :** $\text{Lt}_{x\to 0}\frac{\sin[\cos x]}{1+[\cos x]}=\frac{\sin 1}{2}$

Reason : $x \to 0 \Rightarrow \cos x \to 1$

4. **Assertion :** If $f(x) = x - x^2 + 1$ and $g(x) = \max\{f(t):0 \le t \le x\}$ Then $\int_0^1 g(x)\,dx = \frac{29}{24}$

Reason : f(x) is increasing in (0, ½) and decreasing $\left(\frac{1}{2}, 1\right)$

SECTION – III

Paragraph

For any two events A and B, $P(B/A) = \frac{P(B\cap A)}{P(A)}$

1. A biased die is tossed and the respective probability with various faces to turn up are :-

Face :	1	2	3	4	5	6
Probability :	0.1	0.24	0.19	0.18	0.15	0.14

If an even face turned up, probability that it is face 2 or face 4 is
(a) 0.25 (b) 0.42 (c) 0.75 (d) 0.9

2. Two friends A and B have equal number of daughters. There are three cinema tickets which are to be distributed among the daughters of A and B. The probability that all the tickets were given to daughters of A is 1/20 : Then the no. of daughters of each of them is :-
(a) 4 (b) 5 (c) 6 (d) 3

3. An urn contains 6 white and 4 black balls. A fair die is rolled and the balls equal to the number on top of dice are chosen from the urn. The probability that the balls selected are white is
(a) 1/5 (b) 1/6 (c) 1/7 (d) 1/8

Paragraph

Consider a parabola $y^2 = 32x$ and a point P(2, –8) on it. A circle C_1 touches that parabola at point P.

1. If C_1 passes through the focus, then its equation is
   (a) $y(y + 8) + (x - 2)(x - 8) = 0$
   (b) $x^2 + y^2 - 6x + 5y + 40 = 0$
   (c) $x^2 + y^2 - 2x + 17y + 72 = 0$
   (d) $x^2 + y^2 - 14x + 11y + 48 = 0$

2. If tangent at vertex of parabola touches C_1, then its equation can be given by—
   (a) $(x - 2)^2 + (y - 8)^2 + 4(\sqrt{5} - 2)(2x + y + 4) = 0$
   (b) $(x - 2)^2 + (y + 8)^2 + 4(\sqrt{5} + 2)(2x + y + 4) = 0$
   (c) $(x - 2)^2 + (y + 8)^2 + 2(\sqrt{5} + 2)(2x + y + 4) = 0$
   (d) $(x - 2)^2 + (y + 8)^2 + 4(\sqrt{5} - 2)(2x + y + 4) = 0$

3. If C_1 touches the parabola again at another point, then its equation is:-
   (a) $(x - 2)^2 + y^2 - 64 = 0$
   (b) $x^2 + y^2 - 16x - 36 = 0$
   (c) $x^2 + y^2 - 36x + 4 = 0$
   (d) $x^2 + y^2 - 24x - 20 = 0$

SECTION IV

---

1. (a) If (1,4) is the centroid of a triangle and the coordinates of any two vertices be(4 –8) and (–9, 7), then twice the area of the triangle is (p) 5

   (b) The distance of the point (2, 5) from the line $3x + y + 4 = 0$ measured parallel to the line $3x - 4y + 8 = 0$ is (q) 33

(c) The value of m for which the lines $3x + y + 2 = 0$, $2x - y + 3x = 0$ and $x + my = 3$ are concurrent is — (r) 333

(d) Two vertices of a triangle are (4, – 3) and (–2, 5) and the orthocenter of the triangle is (1, 2). The x-coordinate of the third vertex is — (s) 4

2. (a) If $a^2 - 2a\cos x + 1 = 674$ and $\tan\frac{x}{2} = 7$, the integral value of 'a' is — (p) 18

(b) In a ΔABC, a = 6, b = 3 and cos (A – B) = 4/5. The area of Δ ABC is — (q) 0

(c) If $\sin\propto + \cos\propto = m$, then $\sin \propto + \cos^6\propto = \frac{4-3(m^2-1^2)}{4}$ provided the maximum value of m^2 is — (r) 2

(d) r_1, r_2, r_3 are the radii of the circles drawn on the altitudes MD, ME and MF of ΔMBC, ΔMCA, ΔMAB as diameter, where m is the circumcentre of the acute angled ΔABC. Then the minimum value of $\frac{1}{18}\left(\frac{a^2}{r_1^2}+\frac{b^2}{r_2^2}+\frac{c^2}{r_3^2}\right)$ is — (s) 25

3. (a) $\int_0^{\pi/2}\frac{dx}{4\sin^2x+\cos^2x}$ — (p) 0

(b) $\int_0^{\pi}\frac{\sin nx}{\sin x}dx$, when n is an even integer — (q) π

(c) $2+\int_{\ln 3}^{\ln 4} e^x\frac{\sqrt{e^x-3}}{2-e^x}dx$ — (r) $\pi/4$

(d) $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{4n}{n^2+r^2}$ — (s) $\pi/2$