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AskIITians IIT JEE Maths Test

Code – AM211

Time - One hour

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

A. General :

1. This booklet is your Question paper containing 69 questions.
2. Blank papers, clipboard, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets in any form are not allowed to be carried inside the examination hall.
3. The answer sheet, a machine-readable Objective Response Sheet (ORS), is provided separately.

B. Filling the ORS :

4. On the lower part of the ORS, write in ink, your name, your Registration No. Do not write these anywhere else.
5. Make sure the CODE on the ORS is the same as that on this booklet and put your signature on the ORS affirming that you have verified.
6. Write your Registration No. in ink, provided in the lower part of the ORS and darken the appropriate bubble UNDER each digit of your Registration No. with a good quality HB pencil.

C. Question paper format.

7. The question paper consists of 3 parts (Physics, Chemistry and Mathematics). Each part has 4 sections.
8. Section I contains 6 multiple choice question. Each question has four choices (A), (B), (C) and (D), out of which only one is correct.
9. Section II contains 4 questions. Each question has four choices (A), (B), (C) and (D), out of which one or more choices is correct.
10. Section III contains 4 questions. Each question contains Statement -1 (Assertion) and Statement -2 (Reason).
    Bubble (A) if both the statements are TRUE and STATEMENT-2 is the correct explanation of STATEMENT-1.
    Bubble (B) if both the statements are TRUE butSTATEMENT-2 is NOT the correct explanation of STATEMENT-2.
    Bubble (C) if STATEMENT-1 is TRUE and STATEMENT-2 is FALSE.
    Bubble (D) if STATEMENT-1 is FALSE and STATEMENT-2 is TRUE.
11. Section IV contains 3 paragraphs. Based upon each paragraph, three multiple choice questions have to be answered. Each question has four choices (A) (B) (C) (D) out of which only one is correct.

D. Marking Scheme.

12. For each question in Section I, you will be awarded 3 marks if you have darkened only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, minus one (–1) mark will be awarded.
13. For each question in Section II, you will be awarded 4 marks, if you darken only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, (–1) mark will be awarded.
14. For each question in Section III, you will be awarded 3 marks, if you darken only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, (–1) mark will be awarded.
15. For each question in Section IV, you will be awarded 3 marks, if you darken only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, (–1) will be awarded.

Useful Data

Gas Constant	R	$= 8.314\ J\ K^{-1}\ mol^{-1}$	**1 Faraday**	=	96500 Coulomb
		$= 0.0821$ Lit atm $K^{-1}\ mol^{-1}$	**1 calorie**	=	4.2 Joule
		$= 1.987 \approx 2$ Cal $K^{-1}\ mol^{-1}$	**1 Ev**	=	1.6×10^{-1} J
Avogadro's Number	Na	$= 6.023 \times 1023$			
Planck's constant	h	$= 6.625 \times 10^{-34}$ J . s			

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$= 6.625 \times 10^{-27}$ erg . s

Atomic No:	H = 1, D = 1, Li = 3, Na = 11, K = 19, Rb = 37, Cs = 55, F = 9, Ca = 20, He = 20, He = 2, O = 8, Au = 79, Ni = 28, Zn = 30, Cu = 29, Cl = 17, Br = 35, Cr = 24, Mn = 25, Fe = 26, S = 16, P = 15, C = 6, N = 7, Ag = 47.
Atomic Masses:	He = 4, Mg = 24, C = 12, O = 16, N = 14, P = 31, Br = 80, Cu = 63.5, Fe = 56, Mn = 55, Pb = 207, Au = 197, Ag = 108, F = 19, H = 1, Cl = 35.5, Sn = 118.6, Na = 23, D = 2, Cr = 52, K = 39, Ca = 40, Li = 7, Be = 4, Al = 27, S = 32.

SECTION – I

1. The sum of the solutions of the equation
$2\sin^{-1}\sqrt{x^2+x+x1} + \cos^{-1}\sqrt{x^2+x} = 3\pi/2$ is
(a) 0
(b) −1
(c) 1
(d) 2

2. Equation of chord of the circle $x^2 + y^2 - 3x - 4y - 4 = 0$ which passes through the origin such that origin divides it in the ratio 4 : 1 is
(a) x = 0
(b) 24x + 7y = 0
(c) 7x + 24y = 0
(d) 7x − 24y = 0

3. If $\vec{a} + 2\vec{b} + 3\vec{c} = \vec{0}$, then $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is equal to
(a) $6(\vec{b} \times \vec{c})$
(b) $6(\vec{c} \times \vec{a})$
(c) $6(\vec{a} \times \vec{b})$
(d) none of these

4. Minimum area of circle which touches the parabola's $y = x^2 + 1$ and $y^2 = x-1$ is
(a) $\frac{9\pi}{16}$ sq. units
(b) $\frac{9\pi}{32}$ sq. units
(c) $\frac{9\pi}{8}$ sq. units
(d) $\frac{9\pi}{4}$ sq. units

5. Maximum value of $x^2 \ln\frac{1}{x}$ is
(a) 2e
(b) e
(c) $\frac{1}{e}$
(d) $\frac{1}{2e}$

6. Let f : R → R be any function and $g(x) = \frac{1}{f(x)}$. Then g is
(a) onto if f is onto
(b) one-one if f is one-one

(c) continuous if f is continuous
(d) differentiable if f differentiable

SECTION – II

1. If the chord through the points whose eccentric angles are θ and ϕ on the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 passes through a focus, then the value of tan(θ/2) tan (ϕ/2) is
(a) $\frac{e+1}{e-1}$
(b) $\frac{e-1}{e+1}$
(c) $\frac{1+e}{1-e}$
(d) $\frac{1-e}{1+e}$

2. If (5, 12) and (24, 7) are the foci of a conic passing through the origin, then the eccentricity of the conic is
(a) $\frac{\sqrt{386}}{12}$
(a) $\frac{\sqrt{386}}{13}$
(b) $\frac{\sqrt{386}}{25}$
(d) $\frac{\sqrt{386}}{38}$

3. If f(x) = $\frac{(1+x)^{1/x}-e}{x}$, then
(a) $\text{Lt}_{x\to\infty} f(x) = \frac{e}{2}$
(b) $\text{Lt}_{x\to\infty} f(x) = \frac{-e}{2}$
(c) $\text{Lt}_{x\to\infty} f(x)$ > 1
(d) $\text{Lt}_{x\to\infty} f(x)$ < −1.

4. If $\sum_{r=1}^{n} r(r+1) = \frac{(n+a)(n+b)(n+c)}{3}$ where a < b < c, then
(a) 2b = c
(b) $a^3 - 8b^3 + c^3 = 8abc$
(c) c is prime no.
(d) $(a+b)^2 = 0$

SECTION – III

1. Statement 1 : If f1 (x), $f_2(x)$, ……, $f_9(x)$ are polynomials whose degree $\geq$ 1, where f1 (α) = $f_2(\alpha)$ =…= $f_9(\alpha)$ = and A (x) = $\begin{bmatrix} f_1(x) & f_2(x) & f_3(x) \\ f_4(x) & f_5(x) & f_6(x) \\ f_7(x) & f_8(x) & f_9(x) \end{bmatrix}$ then $\frac{A(x)}{x-\alpha}$ is also a matrix of 3 × 3 whose entries are also polynomials.
   Statement 2 : (x–α) is a factor of polynomial f(x) if f(α) = 0.

2. Statement 1 : Out of 5 tickets consecutively numbered, three are drawn at random, the chance that the numbers on them are in A.P. is $\frac{2}{15}$.
   Statement 2 : Out of (2n + 1) tickets consecutively numbered, three are drawn at random, the chance that the numbers on them are in A.P. is $\frac{3n}{4n^2-1}$.

3. Statement 1 : If x > 0, x ≠ 1, then $\int (log_x{}^e - (log_x{}^e)^2)dx = xlog_x{}^e + c$
   Statement 2 : $\int e^x$ (f(x) + f^1(x)) dx = e^x f(x) + C and e^t = x if t = ln x.

4. Statement 1 : A point on the straight line 2x + 3y – 4z = 5 and 3x – 2y + 4z = 7 can be determined by taking x = k and then solving the two equations for y and z, where k is any real number.
   Statement 2 : If $C' \neq kc$, then the straight line ax + by + cz + d = 0, kax + kby + c'z + d' = 0, does not intersect the plane z = α, where α is any real number.

SECTION – IV

Paragraph

Consider ΔABC in Argand plane, Let A(0), B(1) and C(1 + i) be its vertices and M be the midpoint of CA. Let z be a variable complex number in the plane. Let u be another variable complex number defined as u = z^2 + 1.

1. Locus of u, when z is on BM is
   (a) circle
   (b) parabola
   (c) ellipse
   (d) hyperbola

2. Axis of locus of y, where z is on BM is
   (a) real-axis
   (b) imaginary axis
   (c) $z + \bar{z} = 2$
   (d) $z - \bar{z} = 2i$

3. Directrix of locus of u, when z is on BM is
   (a) real axis
   (b) imaginary axis
   (c) $z + \bar{z} = 2$
   (d) $z - \bar{z} = 2i$

Paragraph

P is a variable point on the line L = 0. Tangents are drawn to the circle $x^2 + y^2 = 4$ from P to touch it at Q and R. The parallelogram PQSR is completed.

1. If $L = 2x + y - 6 = 0$, then the locus of circumcircle of ΔPQR is
   (a) $2x - y = 4$
   (b) $2x + y = 3$
   (c) $x - 2y = 4$
   (d) $x + 2y = 3$

2. If P = (6, 8), then the area of ΔQRS is
   (a) $\frac{6^{3/2}}{25}$ sq. units
   (b) $\frac{24^{3/2}}{25}$ sq. units
   (c) $\frac{48\sqrt{6}}{25}$ sq. units
   (d) $\frac{196\sqrt{6}}{25}$ sq. units

3. If P = (3, 4), then coordinates of S is
   (a) $\left(\frac{-46}{25}, \frac{-63}{25}\right)$
   (b) $\left(\frac{-51}{25}, \frac{-68}{25}\right)$
   (c) $\left(\frac{-46}{25}, \frac{-68}{25}\right)$
   (d) $\left(\frac{-68}{25}, \frac{-51}{25}\right)$

Paragraph

Integral $\int_a^b f(x)dx$ can be represented as a limit of a sum of infinite series

$\int_a^b (x)dx = \text{Lt}_{x\to\infty} \sum_{r=na+c}^{nb+c} \frac{1}{n} f\left(\frac{r}{n}\right)$, where na + c $\le$ r $\le$ nb + c; n $\in$ N, c $\in$ R and any limit of sum of series of same form can be changed to definite integral by replacing

(i) $\underset{x\to\infty}{\text{Lt}} \sum \to \int$

(ii) $\frac{1}{n} \to dx$

(iii) $\frac{r}{n} \to x$

(iv) Lower limit = $\underset{n\to\infty}{\text{Lt}} \left(\frac{r}{n}\right)_{min} = \underset{n\to\infty}{\text{Lt}} \left(\frac{na+c}{n}\right) = a$

(v) Upper limit = $\underset{n\to\infty}{\text{Lt}} \left(\frac{r}{n}\right)_{max} = \underset{n\to\infty}{\text{Lt}} \left(\frac{nb+c}{n}\right) = b$

1. Find the value of $\underset{x\to\infty}{\text{Lt}} \left(\frac{n}{(n+1)\sqrt{2n+1}} + \frac{n}{(n+2)\sqrt{2(2n+2)}} + \frac{n}{(n+3)\sqrt{3(2n+3)}} + \ldots + \frac{n}{2n\sqrt{3}}\right)$
   (a) $\frac{\pi}{3}$
   (b) $\frac{\pi}{2}$
   (c) $\frac{\pi}{4}$
   (d) none of these

2. The n^th term of the corresponding series of $\int_0^1 tan^{-1}x dx$ is
   (a) $\frac{\pi}{4n}$
   (b) $\frac{1}{n}$ $\tan^{-1}$ (n–1)
   (c) $\frac{\pi}{2n}$
   (d) $\tan^{-1}$ n

3. $\text{Lt}_{n\to\infty} \sum_{r=0}^{2n-1} \frac{1}{n} sec^2\left(\frac{r}{n}\right)$ = ?
   (a) sec 2
   (b) tan 2
   (c) $\sec^2$ 2
   (d) not defined