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AskIITians IIT JEE Maths Test

Code – AC214

Time - One hour

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

A. General :

1. This booklet is your Question paper containing 69 questions.
2. Blank papers, clipboard, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets in any form are not allowed to be carried inside the examination hall.
3. The answer sheet, a machine-readable Objective Response Sheet (ORS), is provided separately.

B. Filling the ORS :

4. On the lower part of the ORS, write in ink, your name, your Registration No. Do not write these anywhere else.
5. Make sure the CODE on the ORS is the same as that on this booklet and put your signature on the ORS affirming that you have verified.
6. Write your Registration No. in ink, provided in the lower part of the ORS and darken the appropriate bubble UNDER each digit of your Registration No. with a good quality HB pencil.

C. Question paper format.

7. The question paper consists of 3 parts (Physics, Chemistry and Mathematics). Each part has 4 sections.
8. Section I contains 6 multiple choice question. Each question has four choices (A), (B), (C) and (D), out of which only one is correct.
9. Section II contains 4 questions. Each question has four choices (A), (B), (C) and (D), out of which one or more choices is correct.
10. Section III contains 4 questions. Each question contains Statement -1 (Assertion) and Statement -2 (Reason).
    Bubble (A) if both the statements are TRUE and STATEMENT-2 is the correct explanation of STATEMENT-1.
    Bubble (B) if both the statements are TRUE butSTATEMENT-2 is NOT the correct explanation of STATEMENT-2.
    Bubble (C) if STATEMENT-1 is TRUE and STATEMENT-2 is FALSE.
    Bubble (D) if STATEMENT-1 is FALSE and STATEMENT-2 is TRUE.
11. Section IV contains 3 paragraphs. Based upon each paragraph, three multiple choice questions have to be answered. Each question has four choices (A) (B) (C) (D) out of which only one is correct.

D. Marking Scheme.

12. For each question in Section I, you will be awarded 3 marks if you have darkened only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, minus one (–1) mark will be awarded.
13. For each question in Section II, you will be awarded 4 marks, if you darken only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, (–1) mark will be awarded.
14. For each question in Section III, you will be awarded 3 marks, if you darken only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, (–1) mark will be awarded.
15. For each question in Section IV, you will be awarded 3 marks, if you darken only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, (–1) will be awarded.

Useful Data

Gas Constant	R	= 8.314 J K^{-1} mol^{-1}	**1 Faraday**	=	96500 Coulomb
		= 0.0821 Lit atm K^{-1} mol^{-1}	**1 calorie**	=	4.2 Joule
		= 1.987 ≈ 2 Cal K^{-1} mol^{-1}	**1 Ev**	=	1.6 × 10^{-1} J
Avogadro's Number	Na	= 6.023 × 1023			
Planck's constant	h	= 6.625 × 10^{-34} J . s			

$= 6.625 \times 10^{-27}$ erg . s

Atomic No: H = 1, D = 1, Li = 3, Na = 11, K = 19, Rb = 37, Cs = 55, F = 9, Ca = 20, He = 20, He = 2, O = 8, Au = 79, Ni = 28, Zn = 30, Cu = 29, Cl = 17, Br = 35, Cr = 24, Mn = 25, Fe = 26, S = 16, P = 15, C = 6, N = 7, Ag = 47.

Atomic Masses: He = 4, Mg = 24, C = 12, O = 16, N = 14, P = 31, Br = 80, Cu = 63.5, Fe = 56, Mn = 55, Pb = 207, Au = 197, Ag = 108, F = 19, H = 1, Cl = 35.5, Sn = 118.6, Na = 23, D = 2, Cr = 52, K = 39, Ca = 40, Li = 7, Be = 4, Al = 27, S = 32.

SECTION – I

1. Range of the function $f(x) = \cos^{-1}(-\{x\})$, where {.} is fractional part function, is
   (a) $\left(\frac{\pi}{2}, \pi\right)$
   (b) $\left(\frac{\pi}{2}, \pi\right)$
   (c) $\left[\frac{\pi}{2}, \pi\right)$
   (d) $\left(0, \frac{\pi}{2}\right]$

2. P(a, b) is a point in the first quadrant. If the two circles which pass through P and touch both the coordinate axes cut at right angles, then
   (a) $a^2 - 6ab + b^2 = 0$
   (b) $a^2 + 2ab - b^2 = 0$
   (c) $a^2 - 4ab + b^2 = 0$
   (d) $a^2 - 8ab + b^2 = 0$

3. If $\vec{a_1}$, $\vec{a_2}$ and $\vec{a_3}$ are non coplanar vectors and $(x + y - 3)\vec{a_1} + (2x - y + 2)\vec{a_2} + (2x + y + \lambda)\vec{a_3} = 0$ holds for some 'x' an 'y', then λ is
   (a) $\frac{7}{3}$
   (b) 2
   (c) $\frac{-10}{3}$
   (d) $\frac{5}{3}$

4. Normals at three points P, Q, R at the parabola $y^2 = 4ax$ meet in a point A and S be its focus, if $|SP|.|SQ|.|SR| = \lambda(SA)^2$, then, λ = ?
   (a) a^3
   (b) a^2
   (c) a
   (d) 1

5. if $f(x) = x^3 - x^2 + 100x + 1001$, then
   (a) f(2000) > f(2001)

(b) $f\left(\frac{1}{1999}\right) > f\left(\frac{1}{2000}\right)$
(c) $f(x+1) > f(x-1)$
(d) $f(3x - 5) > f(3x)$

6. A function g defined for all real $x > 0$ satisfied $g(1) = 1$, $g'(x^2) = x^3$ for all $x > 0$, then $g(4)$ equals
(a) $\frac{13}{3}$
(b) 3
(c) $\frac{67}{5}$
(d) none of these

7. Number of distinct normal lines that can be drawn to ellipse $\frac{x^2}{169} + \frac{y^2}{25} = 1$ from the point P(0, 6) is
(a) 1
(b) 2
(c) 3
(d) 4

8. The function, $f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$ is
(a) an odd function
(b) an even function
(c) neither an odd nor an even function
(d) a periodic function.

9. If a_1, a_2, a_3, a_4, a_5 are in H.P., then $a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5$ is equal to
(a) $2a_1 a_5$
(b) $3a_1 a_5$
(c) $4a_1 a_5$
(d) -4

SECTION – II

1. Statement 1: $\rightarrow$ For a singular matrix A, if AB= AC $\Rightarrow$ B = C
Statement 2: $\rightarrow$ If $|A| = 0$, then A^{-1} does not exists.

2. Statement 1: $\rightarrow$ $x + y + z = 5$ and $xy + yz + zx = 3$, $(x, y, z \in R)$ then the probability for x is positive only is $\frac{13}{6}$.

Statement 2: $\rightarrow$ If x + y + z = 5 and xy + yz + zx = 3, then maximum and minimum value of x, y, z are same.

3. Statement 1: $\rightarrow \int 2^{\tan^{-1}x}\, d(\cot^{-1}x) = \frac{2^{\tan^{-1}x}}{\ln 2} + C$.

Statement 2: $\rightarrow \frac{d(a^x + c)}{dx} = a^x \ln a$.

4. Statement 1: $\rightarrow$ If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then equation $\vec{r} \times (2\hat{i} - \hat{j} + 3\hat{k}) = 3\hat{i} + \hat{k}$ represents a straight line.

Statement 2: $\rightarrow$ If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then equation $\vec{r} \times (\hat{i} + 2\hat{j} - 3\hat{k}) = 2\hat{i} - \hat{j}$ represents a straight line.

SECTION – III

Paragraph

The complex slope of a line passing through two points represented by complex numbers z_1 and z_2 is defined by $\frac{z_2 - z_1}{\bar{z}_2 - \bar{z}_1}$ and we shall denote by w. If z_o is complex number and c is a real number, then $\bar{z}_o z + z_o \bar{z} + c = 0$ represents a straight line. Its complex slope is $-\frac{z_o}{\bar{z}_o}$. Now consider two lines

$$\alpha\bar{z} + \bar{\alpha}z + i\beta = 0 \qquad \text{—— (i)}$$

and $$a\bar{z} + \bar{a}z + b = 0 \qquad \text{—— (ii)}$$

where α, β and a, b are complex constants and their complex slopes be denoted by w_1 and w_2 respectively.

1. If the lines are inclined at an angle of 120° to each other, then
   (a) $w_2 \bar{w}_1 = w_1 \bar{w}_1$
   (b) $w_2 \bar{w}_1^{\,2} = w_1 \bar{w}_1^{\,2}$
   (c) $w_1^2 = w_2^2$
   (d) $w_1 + 2w_2 = 0$

2. Which of the following must be true?
   (a) 'a' must be purely imaginary
   (b) β must be purely imaginary
   (c) 'a' must be real
   (d) 'b' must be imaginary

3. If line (i) makes an angle of 45° with real axis, then $(1 + i)\left(-\frac{2\alpha}{\bar{\alpha}}\right)$ is

(a) $2\sqrt{2}$
(b) $2\sqrt{2}\,i$
(c) $2(1 - i)$
(d) $-2(1 + i)$

Paragraph

Consider the function defined by the equation $y^2 - 2y\,e^{\sin^{-1}x} + x^2 - 1 + [x] + e^{2\sin^{-1}x} = 0$, where [x] denotes greatest integer function.

1. The area of the region bounded by the curve and the line $x = -1$ is
   (a) $\pi + 1$
   (b) $\pi - 1$
   (c) $\frac{\pi}{2} + 1$
   (d) $\frac{\pi}{2} - 1$.

2. Line $x = 0$ divided the region mentioned above in two parts. The ratio of area of left hand side of line to right hand side of line is
   (a) $\frac{2+\pi}{\pi}$
   (b) $\frac{2-\pi}{\pi}$
   (c) $\frac{1}{1}$
   (d) $\frac{\pi+2}{2\pi}$

3. Area of the region of curve and line $x = 0$ and $x = \frac{1}{2}$ is
   (a) $\frac{\sqrt{3}}{4} + \frac{\pi}{6}$
   (b) $\frac{\sqrt{3}}{2} + \frac{\pi}{6}$
   (c) $\frac{\sqrt{3}}{4} - \frac{\pi}{6}$
   (d) $\frac{\sqrt{3}}{2} - \frac{\pi}{6}$

SECTION – IV

1. Consider the word "HONOLULU"

Column I		Column II	
(a)	Number of words that can be formed using the letters of the given word in which consonants and vowels are alternative is	(p)	26
(b)	Number of words that can be formed without changing the order of vowels is	(q)	144
(c)	Number of ways in which 4 letters can be selected from the letters of the given word is	(r)	840
(d)	Number of words in which two O's are together but U's are separated is	(s)	900

2.

Column I		Column II	
(a)	In a ΔABC $(a+b+c)(b+c-a) = \lambda bc$, where $\lambda \varepsilon I$, then greatest value of λ is	(p)	3
(b)	In a ΔABC, $\tan A + \tan B + \tan C = 9$. If $\tan^2 A + \tan^2 B + \tan^2 C = k$, then least value of k satisfying is	(q)	$9(3)^{1/3}$
(c)	In a triangle ABC, line joining the circumcentre to the incentre is parallel to BC, then value of Cos B + Cos C is	(r)	1
(d)	If in a ΔABC, $a = 5$, $b = 4$ and $\cos(A-B) = \frac{31}{32}$, then the third side 'c' is equal to	(s)	6

3.

Column I	Column II

(a)	The area of the triangle whose vertices are the points with rectangular Cartesian coordinates (1, 2, 3), (−2, 1, −4), (3, 4, −2) is	(p)	0
(b)	The value of $(\vec{a} \times \vec{b}) . (\vec{c} \times \vec{d})$ $+ (\vec{b} \times \vec{c}) . (\vec{a} \times \vec{d})$ $+ (\vec{c} \times \vec{a}) . (\vec{b} \times \vec{d})$ is	(q)	2
(c)	A square PQRS of side length P is folded along the diagonal PR so that the point Q reaches at Q′ and planes PRQ′ and PRS are perpendicular to one another, the shortest distance between PQ′ and RS is $\frac{KP}{\sqrt{6}}$, then k = ?	(r)	$\frac{\sqrt{1218}}{2}$
(d)	$\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$, $\vec{b} = -\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$, $\vec{c} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{d} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$, then $(\vec{a} \times \vec{b}) . (\vec{c} \times \vec{d}) = ?$	(s)	21