Askiitians' IIT JEE Maths Test

Code – AM205

Time - One hour

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

A. General :

1. This booklet is your Question paper containing 69 questions.
2. Blank papers, clipboard, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets in any form are not allowed to be carried inside the examination hall.
3. The answer sheet, a machine-readable Objective Response Sheet (ORS), is provided separately.

B. Filling the ORS :

4. On the lower part of the ORS, write in ink, your name, your Registration No. Do not write these anywhere else.
5. Make sure the CODE on the ORS is the same as that on this booklet and put your signature on the ORS affirming that you have verified.
6. Write your Registration No. in ink, provided in the lower part of the ORS and darken the appropriate bubble UNDER each digit of your Registration No. with a good quality HB pencil.

C. Question paper format.

7. The question paper consists of 3 parts (Physics, Chemistry and Mathematics). Each part has 4 sections.
8. Section I contains 6 multiple choice question. Each question has four choices (A), (B), (C) and (D), out of which only one is correct.
9. Section II contains 4 questions. Each question has four choices (A), (B), (C) and (D), out of which one or more choices is correct.
10. Section III contains 4 questions. Each question contains Statement -1 (Assertion) and Statement -2 (Reason).
    Bubble (A) if both the statements are TRUE and STATEMENT-2 is the correct explanation of STATEMENT-1.
    Bubble (B) if both the statements are TRUE butSTATEMENT-2 is NOT the correct explanation of STATEMENT-2.
    Bubble (C) if STATEMENT-1 is TRUE and STATEMENT-2 is FALSE.
    Bubble (D) if STATEMENT-1 is FALSE and STATEMENT-2 is TRUE.
11. Section IV contains 3 paragraphs. Based upon each paragraph. Three multiple choice questions have to be answered. Each question has four choices (A) (B) (C) (D) out of which only one is correct.

D. Marking Scheme.

12. For each question in Section I, you will be awarded 3 marks if you have darkened only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, minus one (–1) mark will be awarded.
13. For each question in Section II, you will be awarded 4 marks, if you darken only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, (–1) mark will be awarded.
14. For each question in Section III, you will be awarded 3 marks, if you darken only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, (–1) mark will be awarded.
15. For each question in Section IV, you will be awarded 3 marks, if you darken only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, (–1) will be awarded.

Useful Data

Gas Constant	R	$= 8.314\ J\ K^{-1}\ mol^{-1}$	**1 Faraday**	= 96500 Coulomb
		$= 0.0821$ Lit atm $K^{-1}\ mol^{-1}$	**1 calorie**	= 4.2 Joule
		$= 1.987 \approx 2$ Cal $K^{-1}\ mol^{-1}$	**1 Ev**	= 1.6×10^{-1} J
Avogadro's Number	Na	= 6.023 × 1023		
Planck's constant	h	$= 6.625 \times 10^{-34}$ J . s		

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$= 6.625 \times 10^{-27}$ erg . s

Atomic No: H = 1, D = 1, Li = 3, Na = 11, K = 19, Rb = 37, Cs = 55, F = 9, Ca = 20, He = 20, He = 2, O = 8, Au = 79, Ni = 28, Zn = 30, Cu = 29, Cl = 17, Br = 35, Cr = 24, Mn = 25, Fe = 26, S = 16, P = 15, C = 6, N = 7, Ag = 47.

Atomic Masses: He = 4, Mg = 24, C = 12, O = 16, N = 14, P = 31, Br = 80, Cu = 63.5, Fe = 56, Mn = 55, Pb = 207, Au = 197, Ag = 108, F = 19, H = 1, Cl = 35.5, Sn = 118.6, Na = 23, D = 2, Cr = 52, K = 39, Ca = 40, Li = 7, Be = 4, Al = 27, S = 32.

MATHEMATICS

SECTION I

1. For complex number z, the minimum value of $|z| + |z - \cos\alpha - i\sin\alpha| + |z - 2(\cos\alpha + i\sin\alpha)|$ is
(a) 1 (b) 2
(c) 4 (d) can't say anything

2. The length of the diameter of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, perpendicular to the asymptote of the hyperbola, $\frac{x^2}{16} - \frac{y^2}{9} = 1$ passing through the 1st and 3rd quadrants is
(a) $\frac{100}{\sqrt{431}}$ (b) $\frac{150}{\sqrt{481}}$
(c) $\frac{25}{\sqrt{3}}$ (d) $11\sqrt{2}$

3. If $\vec{p}$ and $\vec{q}$ are unit vectors and α is the acute angle between them, then $3\vec{p} + 2\vec{q}$ is a unit vector for
(a) no value of α (b) exactly one value of α
(c) exactly two values of α (d) more than two values of α

4. Angles A, B and C of a triangle ABC are in AP. If $\frac{b}{c} = \sqrt{\frac{3}{2}}$ then $\angle A$ is equal to:
(a) $\pi/6$ (b) $\pi/4$
(c) $5\pi/12$ (d) $\pi/2$

5. If $\frac{1+a}{3}$ and $\frac{1-a}{4}$ are the probabilities of occurrence of two mutually exclusive events, then
(a) $-1 \le a \le 1$ (b) $-7 \le a \le 5$
(c) $-1 \le a \le 2$ (d) $-4 \le a \le 1$

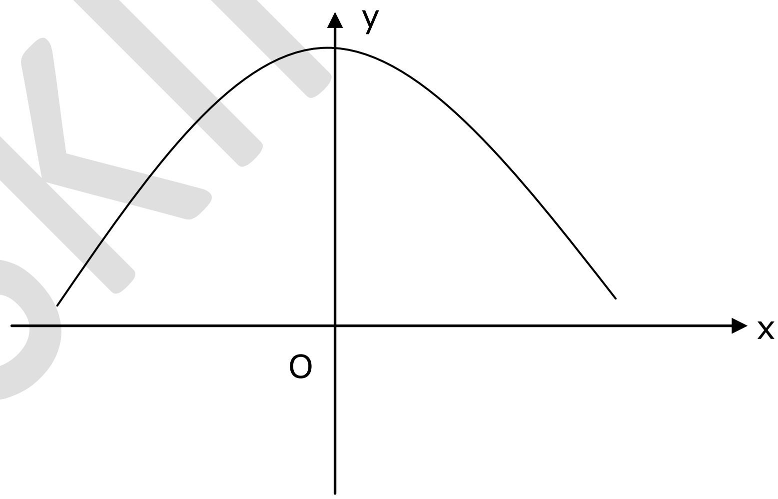
6. $\mathrm{Lt}_{n\to\infty} \frac{1^4 + 2^4 + 3^4 + \ldots\ldots + (2n)^4}{n^5}$ equals
(a) 1/5 (b) 1/6
(c) 32/5 (d) 32/3

SECTION II

1. If a variable tangent of circle $x^2 + y^2 = 1$ intersects the ellipse $x^2 + 2y^2 = 4$ at points P and Q, then the locus of the point of intersection of tangents at P and Q is
   (a) a parabola with latus rectum = 4
   (b) a parabola with focus as (2, 3)
   (c) an ellipse with eccentricity $\frac{\sqrt{3}}{2}$
   (d) an ellipse with eccentricity greater than ½.

2. A plane is such that it passes through the line z = 2; x = -y and contains a point whose distance from yz, zx, xy planes are 3, 4, 5 respectively, then the length of the perpendicular from origin to the plane is
   (a) $\frac{1}{\sqrt{20}}$ (b) $\frac{1}{\sqrt{60}}$
   (c) $\frac{14}{\sqrt{67}}$ (d) $\frac{3}{\sqrt{61}}$

3. Graph of a function f(x) is given below. Which of the given differential equations may have y = f(x) as a solution?



   (a) $\frac{dy}{dx} = 1 + xy$ (b) $\frac{dy}{dx} = -2xy$
   (c) $\frac{dy}{dx} = 1 - 2xy$ (d) none of these

4. If $P(x) = ax^2 + bx + c$ satisfy $|P(x)| \leq 1 \ \forall \ x \in [0, 1]$; $a, b \geq 0$ and $|P'(0)|_{max} = \alpha$ then,

(a) $\alpha = 8$
(b) $\alpha = 2$
(c) $\int_0^1 |P(x)|\, dx = \frac{1}{2}$, when |P′(0)|is maximum
(d) $\int_0^1 |P(x)|\, dx = \alpha - 2$, when |P′(0)|is maximum

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SECTION III

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1. Statement 1: In ΔABC, $(a+b+c)(b+c-a) = \lambda\, bc$ is possible if $0 < \lambda < 4$.
   Statement 2: $-1 < \cos\theta < 1$ where 'θ' is an angle of triangle.

2. Statement 1: The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{3}$ and $\frac{x-1}{-3} = \frac{y-4}{2} = \frac{z-5}{1}$ are skew lines.
   Statement 2: Two non parallel non intersecting lines are skew lines.

3. Statement 1: Normal drawn at a fixed point $P(t_1)$, $t_1 \neq 0$ on the parabola $y^2 = 4ax$ again intersects the parabola at point t_2 for all non zero real values of t_2.
   Statement 2: Normal drawn at a point $P(t_1)$, $t_1 \neq 0$, on the parabola $y^2 = 4ax$ again intersects the parabola at the point t_2, where $t_2 = -t_1 - \frac{2}{t_1}$.

4. Statement 1: Let m, n, a, b and c are non-zero real numbers such that a, b, c are in H.P., then $\frac{a}{m+na}, \frac{b}{m+nb}, \frac{c}{m+nc}$ are also in H.P.
   Statement 2: If a, b, c are in G.P., then $a - \frac{b}{2}, \frac{b}{2}, c - \frac{b}{2}$ are in H.P.

SECTION IV

Paragraph

A non negative differentiable function f(x) is defined on the interval [0, 1] with f(1) = 0. For each $a \in (0, 1)$, the line x = a divides the area bounded by y = f(x) and the coordinate axes in two parts. The area bounded on the left (having y-axis as one boundary) is denoted by A and other area is denoted by B. It is known that A – B = 2f(a) + 3a + b, ∀ a ∈ (0, 1), where b is a constant independent of a.

1. The function satisfies the differential equation
(a) $\frac{dy}{dx} - 2y = -3$
(b) $\frac{dy}{dx} - y = -3/2$
(c) $\frac{dy}{dx} - 3y = \frac{-5}{2}$
(d) none of these

2. The equation of the function is
(a) $f(x) = \frac{3}{2}(1 - e^{x-1})$
(b) $f(x) = \frac{5}{2}(1 - e^{x-1})$
(c) $f(x) = \frac{5}{3}(1 - e^{x-1})$
(d) none of these

3. The value of b is
(a) $\frac{5}{3e} - 3$
(b) $\frac{5}{2e} - 3$
(c) $\frac{3}{2e} - 3$
(d) none of these

Paragraph

$(1 + x)^n = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_nx^n$, where n is a +ve integer

1. $a_0 - a_2 + a_4 - a_6 + \ldots\ldots$ is equal to
(a) $2^{\frac{n}{2}} \cos\frac{n\pi}{4}$
(b) $2^{\frac{n+2}{2}} \cos\frac{n\pi}{4}$
(c) $2^n \cos\frac{n\pi}{4}$
(d) $2^{n+1} \cos\frac{n\pi}{4}$

2. $a_1 - a_3 + a_5 - a_7 + \ldots\ldots$ is equal to
(a) $2^{\frac{n}{2}} \cos\frac{n\pi}{4}$
(b) $2^{\frac{n}{2}+2} \sin\frac{n\pi}{4}$
(c) $2^{\frac{n}{2}} \sin\frac{n\pi}{4}$
(d) $2^{\frac{n+2}{2}} \sin\frac{n\pi}{4}$

3. $a_0 + a_4 + a_8 + \ldots\ldots$ is equal to
(a) $2^n + 2^{n-1} \cos\frac{n\pi}{4}$
(b) $2^{n-2} + 2^{\frac{n}{2}-1} \cos\frac{n\pi}{4}$
(c) $2^n\left(1 + \cos\frac{n\pi}{4}\right)$
(d) $2^{n-1}\left(1 + \cos\frac{n\pi}{4}\right)$

Paragraph

From Coordinate Geometry we know that if the origin is shifted to the point (h,k) without rotating the coordinate axes, then coordinates of a point P1 from old system to new system are given by the transformation

$$x = X + h \qquad \text{and} \qquad y = Y + k$$

where point P has coordinates (x, y) and (X, Y) respectively referred to old and new axes. Now consider a conic in two dimensional system with its equation given by

$$f(x, y) = x^2 - y^2 - 2x + 4y - 12 = 0$$

1. Eccentricity of the conic f(x, y) = 0 is
   (a) 1 (b) $\sqrt{3}$
   (c) $\sqrt{2}$ (d) less than 1

2. The locus of the point of intersection of the perpendicular tangents to f(x, y) = 0 is
   (a) $x^2 + y^2 - 2x - 2y - 4 = 0$ (b) $x^2 + y^2 = 4$
   (c) $x^2 + y^2 - 2x - 2y - 14 = 0$ (d) none of these

3. Number of real tangents which can be drawn to the curve f(x, y) = 0 from the point (5, 6) is
   (a) 1 (b) 2
   (c) 0 (d) 3